Ford Fulkerson and Min Cut Algo Examples

Ford-Fulkerson Algo:

```
MaxFlow(G):

    // Initalize
    Set f(e) = 0 ∀ e ∈ G

    // While there is an s-t path in Gf
    While P = FindPath(s, t, Residual(G,f)) ≠ None:
        f = Augment(f, P)
        UpdateResidual(G, f)

    return f
```

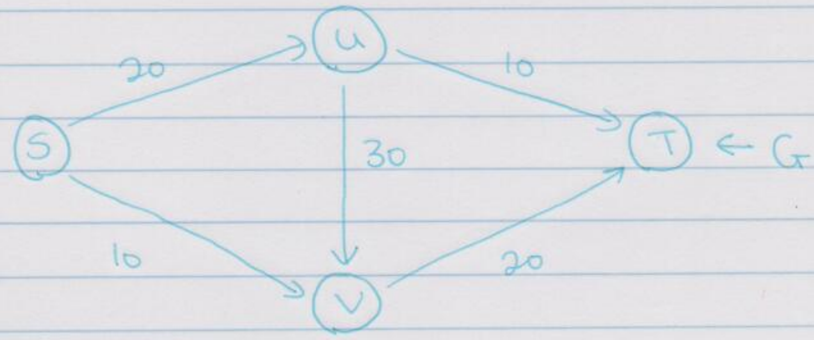
Min-Cut Algo:

1. Run F-F to find a max flow f
2. Construct its residual graph G_f
3. Let A^* be the set of nodes reachable from s in G_f
4. Then $(A^*, V \setminus A^*)$ is a min cut

   Note: We define the cut in G.

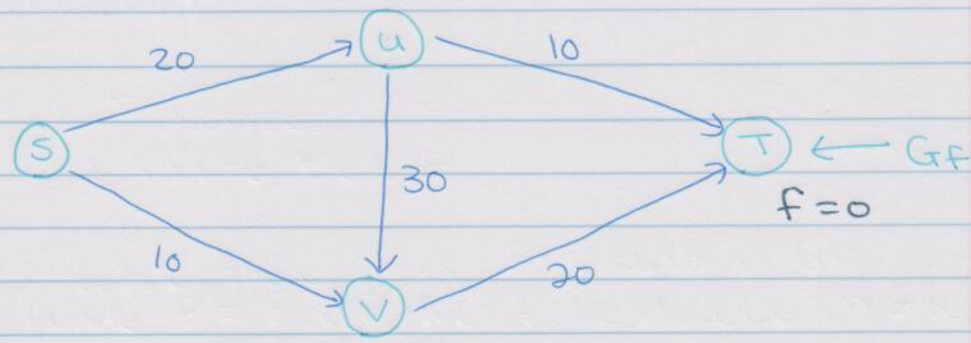
Examples:

1. Find the max-flow and min cut for G:



Soln:

1. Create G_F

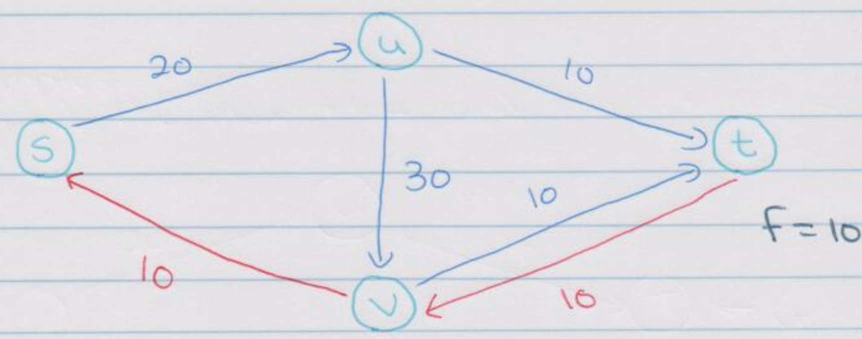


Note: Edges in blue are forward edges.
Edges in red are reverse edges.

2. Find a s-t path in G_f, if one exists.

I'll use the path $(s,v) \rightarrow (v,t)$.
The bottleneck of this path is 10.

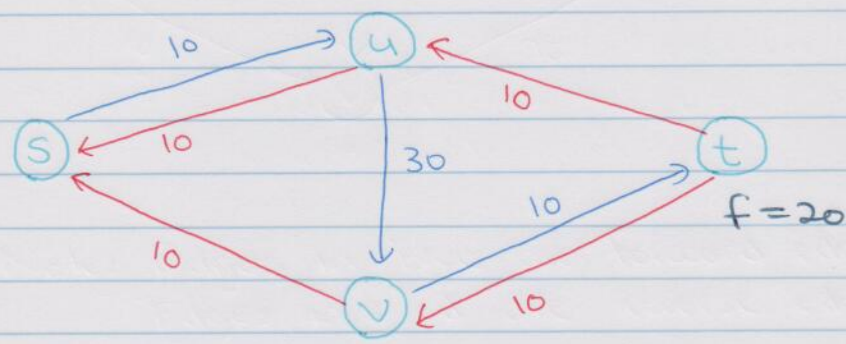
3. Update G_F.



4. Find an s-t path in Gf, if one exists

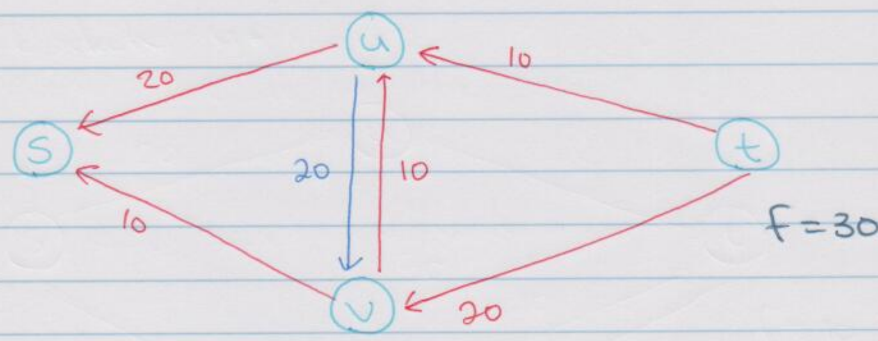
I'll use the path (s,u) → (u,t).
The bottleneck is 10.

5. Update Gf



6. Find an s-t path in Gf, if one exists.

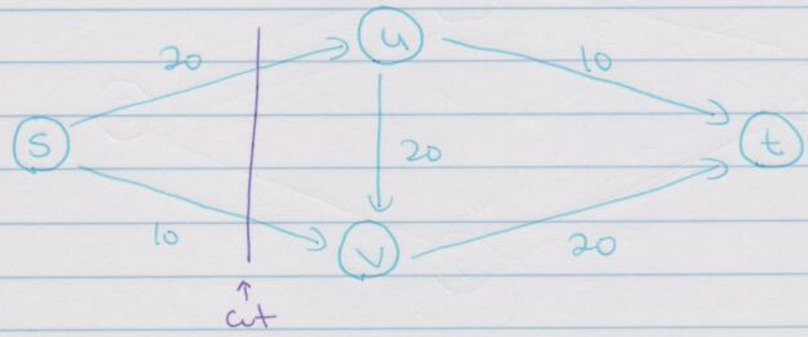
I'll use the path (s,u) → (u,v) → (v,t).
The bottleneck is 10.



There's no more s-t path, so we stop.
The max flow is 30.

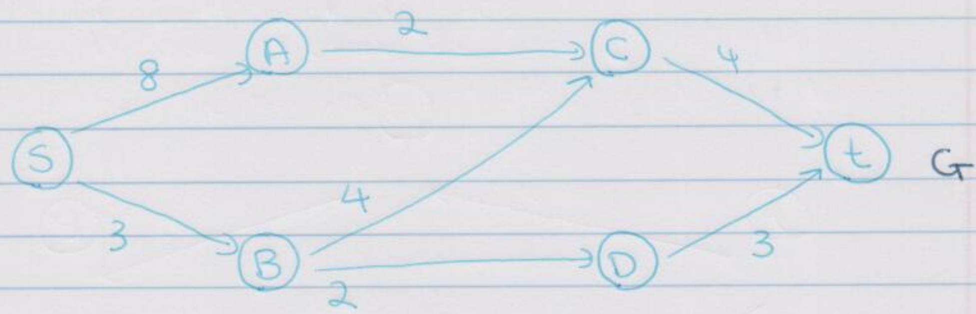
I'll Find the min cut now.
Since the only node reachable from S in Gf is S, $A^* = \{S\}$ and $B^* = \{V \setminus A\}$

Hence, in G, the cut would look like this:

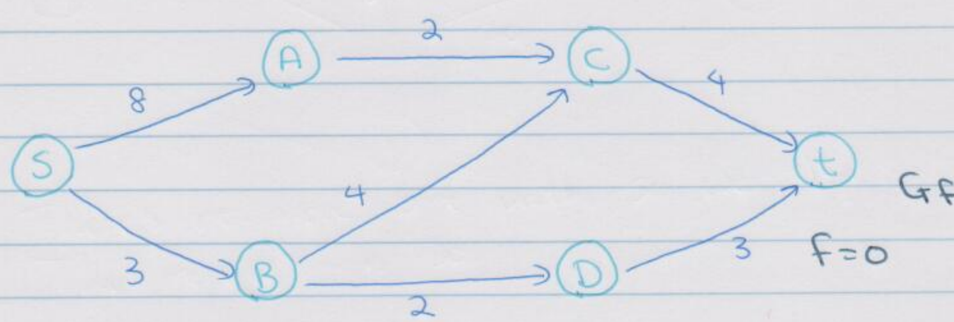


The capacity of the cut is 30.

2. Find the max flow and min cut of G

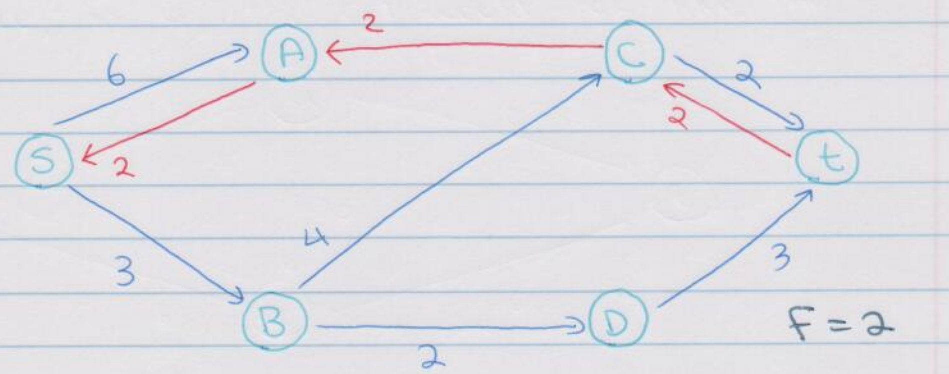


Soln:
1.



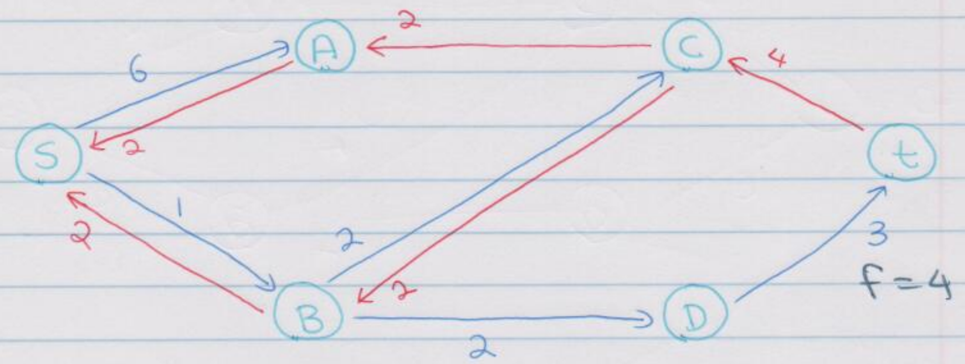
2. I'll use the s-t path $(s,A) \to (A,C) \to (C,t)$.
The bottleneck is 2.

3.

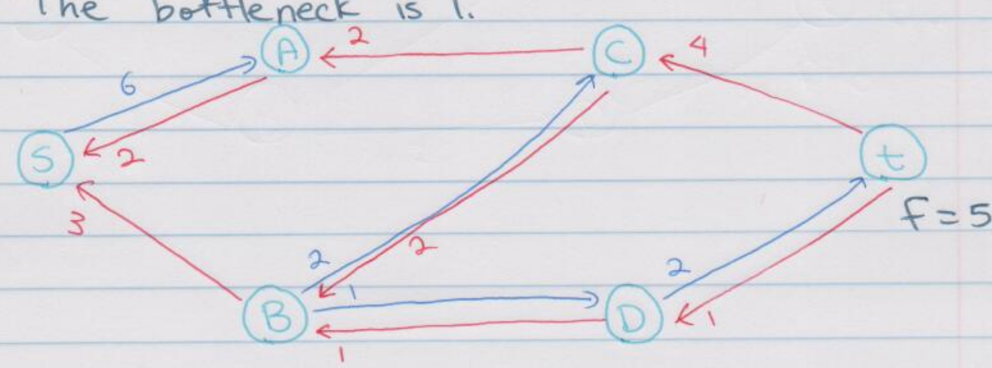


4. I'll use the s-t path $(s,B) \to (B,C) \to (C,t)$.
The bottleneck is 2.

5.



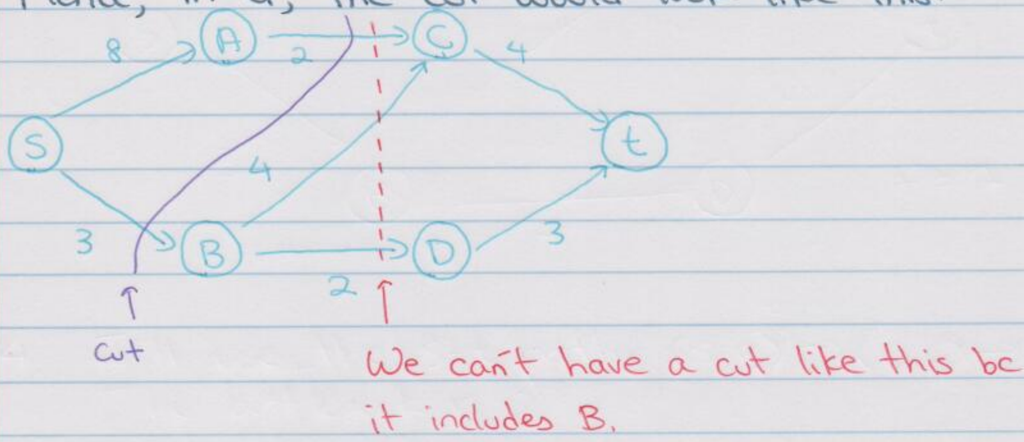
6. I'll use the s-t path (s,B), (B,D), (D,t).
The bottleneck is 1.



Since there's no more s-t path, the max flow is 5.

$A^* = \{S, A\}$ ← Only S and A are reachable from
$B^* = \{V \setminus A\}$ S in G_f.

Hence, in G, the cut would look like this:



∴ The min cut capacity is 5.